Grounding line migration captured by flowline ice sheet models

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Surface mass balance: \( \text{SMB} = \text{Precipitation} - \text{Evaporation} - \text{Runoff} \)
Total mass balance: \( \text{TMB} = \text{SMB} - \text{Discharge (D)} \)
1 mm sea level rise (SLR) \( \approx 360 \text{ Gt/yr} \)

**GREENLAND ICE SHEET (GrIS)**
2.2 million km\(^2\) (no big ice shelves)
SLR if complete melt \( \approx 7 \text{ m} \)
Mass gain by Precipitation
Mass loss by Runoff and Calving

**ANTARCTIC ICE SHEET (AIS)**
14 million km\(^2\) (8% ice shelves)
SLR if complete melt \( \approx 56 \text{ m} \)
Mass gain by Precipitation
Mass loss by Sub-ice shelf melting and Calving
Recent papers in *Nature*:
Pritchard *et al.* (2012), Hellmer *et al.* (2012)

(UNEP Maps and Graphs; based on material provided by K. Steffen)
... gives overall ice loss (2002-2009)

3 different techniques provide mass balance estimates:

1. Satellite altimetry (ERS, ICESat, IceBridge, Cryosat): best for thinning rates
2. Time-variable gravity (GRACE): best for mass balance
3. Mass budget method, i.e. SMB-D (InSAR-RACMO): best for ice dynamics

GRACE results (Velicogna, 2009):

**GREENLAND ICE SHEET (GrIS)**
- Mass budget: $-230 \pm 33 \text{ Gt/yr}$
  $\approx 0.6 \text{ mm/yr SLR}$
- Acceleration: $-30 \pm 11 \text{ Gt/yr}^2$

**ANTARCTIC ICE SHEET (AIS)**
- Mass budget: $-143 \pm 73 \text{ Gt/yr}$
  $\approx 0.4 \text{ mm/yr SLR}$
- Acceleration: $-26 \pm 14 \text{ Gt/yr}^2$
Contributions to sea level rise (SLR)

The actual 3.3 mm/yr SLR (2003-2010) comes from:

1. Ocean thermal expansion ≈ 1 mm/yr (Cazenave and Llovel, 2010)
2. Ice sheet loss ≈ 1 mm/yr (Velicogna, 2009; Jacob et al., 2012)
3. Glacier loss ≈ 0.4 mm/yr (Jacob et al., 2012) to 1 mm/yr (Meier et al., 2007; Cogley, 2009; Dyurgerov, 2010)
4. Land water storage change?

(Cazenave, 2012)
The concept of ‘marine ice sheet instability’

- Marine ice sheet: bedrock below sea level (e.g. West Antarctica)
- **Grounding line (GL)**: ice sheet / ice shelf boundary
- Instability when GL is located on upward-sloping beds

Ice velocity and mass balance of large basins
(Rignot *et al.*, 2008)

(Vaughan and Arthern, 2007)
Coupling at the grounding line

Velocity profiles:

- **Ice sheet**: Vertical shear and basal friction
- **Grounding line**: Coupling
- **Ice shelf**: Longitudinal stretching
What are the ingredients of a GL model?

1. Physics (full Stokes, higher-order model (HOM), shallow-shelf approximation (SSA), shallow-ice approximation (SIA))
2. Numerics (finite element, finite difference, pseudo-spectral, etc.)
3. Grid type (fixed, moving, adaptive)
4. Grounding line treatment (contact problem, flotation, heuristic rule, margin tracking)
5. Grid size (mesh refinement at GL or not)
Grounding line dynamics

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Physical approximations

Full Stokes

\[ \rho_i \frac{dv}{dt} = \nabla \cdot \tau + \rho_i g \]

- \( \rho_i \): ice density
- \( v \): velocity field
- \( t \): time
- \( \tau \): stress tensor
- \( g \): gravitational acceleration

Approximations

- Higher-order (HOM) (Pattyn, 2003)
- Shallow-shelf (SSA) (MacAyeal, 1992)
- Shallow-ice (SIA) (Hutter, 1983)

Components of the stress tensor
(Greve and Blatter, 2009)
Physical approximations

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**Components of the stress tensor** (Greve and Blatter, 2009)
Grounding line (GL) migration

To accurately capture GL migration in a model:

1. Either resolve the transition zone at very fine resolution
2. Or apply an analytical constraint on the flux across GL

One of the models in this study (Docquier et al., 2011) is based on the second approach, where GL ice flux $q_g$ is calculated as a function of GL ice thickness $h_g$ and other constant parameters (viscosity $A$, densities of ice $\rho_i$ and water $\rho_w$, gravity $g$, basal sliding parameter $C$) (Schoof, 2007):

$$q_g = \rho_i g^{0.75} h_g^{4.75} = \left[ \frac{A(\rho_i g)^4(1 - \rho_i / \rho_w)^3}{64C} \right]^{0.75} h_g^{4.75}$$

This flux constraint can be incorporated in coarse grid models thanks to a heuristic rule (Pollard and DeConto, 2009):

$$q_g > q_i : \quad u_i = q_g / h_i$$

$$q_g < q_i : \quad u_{i+1} = q_g / h_{i+1}$$

where $q_g$ is the analytical ice flux at GL and $q_i$ is the modeled ice flux at the grid point upstream GL, and where $u$ and $h$ are the ice velocity and thickness resp. ($i$ is the grid point upstream GL and $i + 1$ is the grid point downstream GL).
Flowline models

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(Schoof, 2007)

$x$: horizontal distance from the ice divide
$s$: surface elevation
$h$: ice thickness
$b$: bedrock depth below sea level
$x_g$: grounding line (GL) position
$x_c$: calving front position
$u$: horizontal ice velocity

<table>
<thead>
<tr>
<th></th>
<th>FS-AG (Elmer)</th>
<th>SSA-H-FRG</th>
<th>SSA-FIG</th>
<th>SSA-PSMG</th>
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<tbody>
<tr>
<td>Affiliation</td>
<td>Grenoble (Durand et al., 2009)</td>
<td>ULB (Docquier et al., 2011)</td>
<td>ULB</td>
<td>Cambridge</td>
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<tr>
<td>Physics</td>
<td>Full Stokes</td>
<td>SSA (shallow-shelf approx.)</td>
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<td>Finite Element</td>
<td>Finite Difference</td>
<td>Finite Difference</td>
<td>Pseudo-spectral</td>
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<tr>
<td>Grid type</td>
<td>Adaptive Grid</td>
<td>Fixed Regular Grid</td>
<td>Fixed Irregular Grid</td>
<td>Moving Grid</td>
</tr>
<tr>
<td>GL treatment</td>
<td>Contact problem</td>
<td>Heuristic rule (Pollard, 2009)</td>
<td>Flotation</td>
<td>Margin tracking</td>
</tr>
<tr>
<td>Grid size</td>
<td>50 m at GL</td>
<td>10 km everywhere</td>
<td>50 m at GL</td>
<td></td>
</tr>
<tr>
<td>Vertically integrated?</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td></td>
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</table>
Experimental setup

We start from the same initial conditions for all models (buttressing, accumulation, viscosity, bed topography, densities, etc.)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Unit</th>
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<tbody>
<tr>
<td>b</td>
<td>Bed elevation</td>
<td>-x/1000</td>
<td>m</td>
</tr>
<tr>
<td>(\rho_i)</td>
<td>Ice density</td>
<td>900</td>
<td>kg m(^{-3})</td>
</tr>
<tr>
<td>(\rho_w)</td>
<td>Water density</td>
<td>1000</td>
<td>kg m(^{-3})</td>
</tr>
<tr>
<td>g</td>
<td>Gravitational acceleration</td>
<td>9.8</td>
<td>m s(^{-2})</td>
</tr>
<tr>
<td>A</td>
<td>Glen’s law coefficient</td>
<td>1.5 (\times) 10(^{-25})</td>
<td>Pa(^{-3}) s(^{-1})</td>
</tr>
<tr>
<td>n</td>
<td>Glen’s law exponent</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>Basal friction parameter</td>
<td>10(^6)</td>
<td>Pa m(^{-1/3}) s(^{1/3})</td>
</tr>
<tr>
<td>m</td>
<td>Basal friction exponent</td>
<td>1/3</td>
<td></td>
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<tr>
<td>a</td>
<td>Accumulation rate</td>
<td>0.3</td>
<td>m a(^{-1})</td>
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<tr>
<td>C_F</td>
<td>Buttressing parameter</td>
<td>0.4</td>
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X-axis: Horizontal distance from ice divide
Y-axis: Elevation

- **Full Stokes – Adaptive Grid**
- **SSA – Fixed Irregular Grid**
- **SSA – Heuristic – Fixed Regular Grid**
- **SSA – Pseudo-Spectral Moving Grid**
Loss of buttressing

- Buttressing occurs when part of the stress applied by the ocean at the shelf front $\sigma_{\text{front}}$ is taken up by sidewall stress $\tau_{\text{sidewall}}$, and so longitudinal stresses within the shelf $\sigma_{\text{shelf}}$ are reduced at the grounding line.

- Sub-ice shelf melting $\Rightarrow$ Release of buttressing $\Rightarrow$ GL retreat and ice loss

- Price et al. parameterisation for calculating the longitudinal stress at the calving front $\tau_{xx}^C$ as a function of ice thickness $h_c$:

\[
\tau_{xx}^C = C_F \frac{\gamma}{4} h_c
\]

\[
\gamma = (1 - \frac{\rho_i}{\rho_w})\rho_i g
\]

- Buttressing is released by increasing the calving front parameter $C_F$ and the model is run during 200 years

- **GOAL**: evaluate transient response of GL
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X-axis: Time
Y-axis: GL position (l) and migration rate (r)

Models show remarkable similarities regarding changes in GL position

GL retreats by $\approx 80$ km in the most extreme scenario ($C_F=1$)

GL migration rates comparable to observations (e.g. Pine Island Glacier, West Antarctica)

Continuous changes with SSA-PSMG and SSA-FIG

Numerical noise for FS-AG and SSA-H-FRG

‘Single-cell dithering’ (flipping behaviour) for SSA-H-FRG
Rate of surface elevation change

X-axis: Horizontal distance from divide
Y-axis: Time

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FS-AG
Full Stokes-Adaptive Grid

SSA-FIG
SSA-Fixed Irregular Grid

SSA-H-FRG
SSA-Heuristic-Fixed Regular Grid

X-axis: Horizontal distance from divide
Y-axis: Time
Conclusions

- Ice loss in Greenland and Antarctica, especially important in West Antarctic Ice Sheet, which is a marine ice sheet
- **Grounding line migration is a key process controlling marine ice sheet stability**
- Transient response of 4 flowline ice sheet models to a reduction in buttressing
- All models give very similar results (surface geometry and GL migration), with some limitations for SSA-H-FRG (coarse model with heuristic rule)
- Caution with models that prescribe ice flux at GL
Testing buttressing and basal melting with a real glacier (Thwaites Glacier, West Antarctica)

Implementation of the heuristic rule in 3D (ice flux at GL)

Results are part of ice2sea (FP7 European project)
SSA-H-FRG incorporates the analytical solution, so strongly follows the **boundary layer theory (gray curve)**

- Other models show similar behaviour (boundary layer reached after some relaxation)
- For a given ice thickness, ice flux is overestimated by boundary layer theory during transient